Estimation Theory of Temperature Rise on Sliding Contact Surfaces of Differential Mechanisms for Automotive Drivelines

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This report describes a trial to estimate temperature rise on sliding contact surfaces of differential mechanisms for automotive drivelines by using the moving heat source theory. First, theoretical equations of contact force and relative sliding velocity of main sliding contact surfaces are shown. Next, equations for estimating the temperature rise of sliding contact surfaces and oil are derived. In the numerical examples, the temperature rise of main sliding contact surfaces is estimated to understand the tendency. The validity of this estimation theory is also verified by comparing estimated and measured results of oil temperature.

Key Words: *differential mechanism, automotive driveline, sliding contact surface, temperature rise*

1. Introduction

Differential mechanisms (differential gears) are equipped in automotive drivelines so that rotational disparity occurs between the inner and outer drive wheels allowing the vehicle to turn smoothly.

Also, limited slip differentials proactively use the friction force of sliding contact surfaces on gear pairs, gears and housing cases to create limiting force and improve vehicle stability.

On the development and design phase of these differential mechanisms for automotive drivelines, studies on gear specification and load share are important to prevent seizure of the differential mechanism and use friction force efficiently. An estimation theory for the temperature rise of sliding contact surfaces is required for studies to secure differential mechanisms reliability by avoiding seizure.

There are some researches concerning temperature rise on sliding contact surfaces. For example, estimation theory of temperature rise for the heat source which moved at a constant velocity[1)] and the heat source which have three-dimensional distribution[2), 3)] are described. There is also a research that specifically studies the temperature rise of the gears in spur gears through a moving heat source repeated intermittently[4)]. However, there is not a research for the estimation of temperature rise on sliding contact surfaces of differential mechanisms for automotive drivelines.

This report describes a trial to estimate temperature rise on sliding contact surfaces of differential mechanisms for automotive drivelines by applying the moving heat source theory[1)]. First, theoretical equations of contact force and relative sliding velocity of main sliding contact surfaces are derived. Next, based on these theoretical equations and the moving heat source theory, equations for estimating the temperature rise of sliding contact surfaces and oil used to lubricate/cool differential mechanisms are derived. In the numerical examples, the temperature rise of main sliding contact surfaces is estimated to understand tendency. The validity of this estimation theory is also verified by comparing estimated and measured results for oil temperature.

2. Estimation Theory of the Temperature Rise of Sliding Contact Surfaces and Oil

Schematic view of a differential mechanism for an automotive driveline is shown in **Fig. 1**. Differential mechanisms mainly consist of a housing case, pin, pinion and side gear. In this chapter, the contact between the pin and pinion, the engagement between the side gear and pinion are focused as the main sliding contact surfaces. Theoretical equations for contact force and relative sliding velocity are derived. And also the moving heat source theory is applied to estimate the temperature rise of each sliding contact surface. In addition, an estimation theory is examined for the temperature rise of oil, which lubricates and cools sliding contact surfaces of differential mechanisms.

2. 1 Contact Force and Relative Sliding Velocity of Sliding Contact Surfaces

2. 1. 1 Contact Force

Considering the balance of torque in the pin of **Fig. 1**, contact force of the pin and pinion F_{pp} could be expressed by the following equation:

$$F_{pp} = \frac{T}{k\, r_{ms}} \tag{1}$$

Here, T is the input torque to the differential mechanism, k is the number of pinions and r_{ms} is the radius of the side gear engagement. Also, assuming that differential motion occurs in the differential mechanism, and the side gear (L) in **Fig. 1** as driven surface, while side gear (R) as drive surface, equation (1) can also be expressed by the following equation:

$$F_{up}^{L} + F_{up}^{R} = \frac{T}{k\, r_{ms}} \tag{2}$$

Here, F_{up}^{L} and F_{up}^{R} are the rotational direction element of tooth surface normal force of the side gear and the pinion. Subscripts L and R indicate driven and drive surface respectively and u means rotational direction.

Also, the following equation is established from the balance of moment in the pinion at the time of differential motion being created.

$$r_{mp}F_{up}^{R} = r_{mp}F_{up}^{L} + \mu' r_{pp}\left(F_{up}^{L} + F_{up}^{R}\right) + \mu'' r'_{hp}\left(F_{sp}^{L} + F_{sp}^{R}\right) \tag{3}$$

Here, r_{mp} is the radius of pinion engagement, r_{pp} is the pin radius, r'_{hp} is the pinion and housing case contact face effective radius, while μ' and μ'' are the friction coefficients between the pin and pinion and between the pinion and housing case. The subscript s indicates axial direction.

Furthermore, the forces in each direction used on the pinion of the side gear engagement portion are given respectively in the following relational expressions.

$$\begin{aligned} F_{up}^{L} &= F_{np}^{L}\left(\cos\alpha - \mu\sin\alpha\right) \\ F_{sp}^{L} &= F_{np}^{L}\left(\sin\alpha + \mu\cos\alpha\right)\sin\delta_p \end{aligned} \tag{4}$$

$$\begin{aligned} F_{up}^{R} &= F_{np}^{R}\left(\cos\alpha + \mu\sin\alpha\right) \\ F_{sp}^{R} &= F_{np}^{R}\left(\sin\alpha - \mu\cos\alpha\right)\sin\delta_p \end{aligned} \tag{5}$$

Here, F_{np}^{L} and F_{np}^{R} are tooth surface normal force of the side gear and the pinion. α is the pressure angle, μ is the friction coefficient between the side gear and pinion, δ_p is the pinion cone angle, and the subscript r is radial direction. Tooth surface normal forces F_{np}^{L} and F_{np}^{R} caused by side gear and pinion engagement can be found by substituting equations (2) and (3) with (4) and (5), making them simultaneous, and solving them.

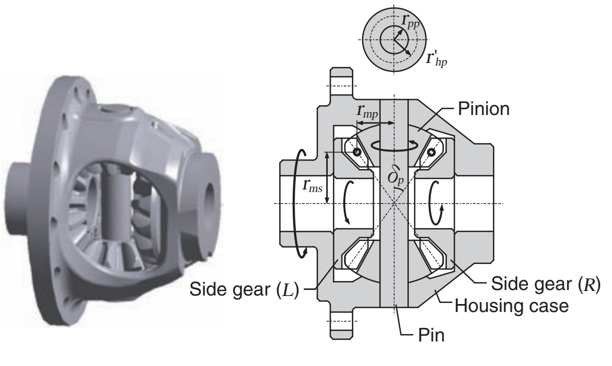

Fig. 1 Schematic view of a differential mechanism for automotive drivelines

2. 1. 2 Relative Sliding Velocity

In this paragraph, the relative sliding velocity is shown on the contact between the pin and pinion as well as the engagement of the side gear and pinion. First, the relative sliding velocity on the contact portion between the pin and pinion v_{pp} is expressed in the following equation by considering the number of teeth.

$$v_{pp} = r_{pp}\frac{\Delta n}{2}\frac{2\pi}{60}\frac{z_s}{z_p} \tag{6}$$

Here, Δn is the differential rotational speed, while z_s and z_p are the number of teeth on the side gear and pinion.

Considering the engagement of equivalent spur gear pair as shown in **Fig. 2**, the relative sliding velocity on the engagement of the side gear and pinion v_{sp} is expressed by the following equation using the distance from the engagement point and pitch point w.

$$v_{sp} = w\frac{\Delta n}{2}\frac{2\pi}{60}\frac{z_s - z_p}{z_p} \tag{7}$$

2. 2 Temperature Rise of Sliding Contact Surfaces

A sliding contact model between the pin and pinion is shown in **Fig. 3**. Assuming line contact and applying the contact force and relative sliding velocity obtained in the previous section, based on the moving heat source theory, the theoretical equation for temperature rise θ_{pp} (X_{pp}, Z_{pp}) of the sliding contact surface at the pinion inner periphery can be expressed by the following equation:

$$\theta_{pp}\left(X_{pp}, Z_{pp}\right) = \frac{2R_{pp}q_{pp}K_p}{\pi k_p v_{pp}}\int_{X_{pp}-L_{pp}}^{X_{pp}+L_{pp}} e^{-u}\mathrm{K}_0\left[\sqrt{Z_{pp}^{\,2}+u^2}\right]du \tag{8}$$

$$q_{pp} = \frac{\mu' F_{pp} v_{pp}}{4 l_{pp} m_{pp}} \tag{9}$$

$$L_{pp} = \frac{v_{pp} l_{pp}}{2K_p},\quad X_{pp} = \frac{v_{pp}X}{2K_p},\quad Z_{pp} = \frac{v_{pp}Z}{2K_p} \tag{10}$$

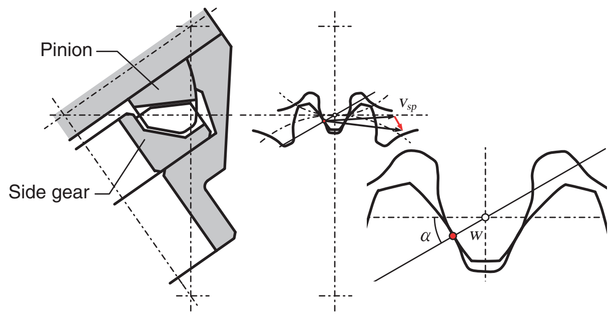

Fig. 2 Engagement of equivalent spur gear pair

Here, R_{pp} is the distribution ratio of the heat to the pinion, K_p and k_p are the thermal diffusivity and thermal conductivity of the pinion, K_0 is the modified Bessel function, $2l_{pp}$ and $2m_{pp}$ are the contact width and contact length, while x and z are the Cartesian coordinate axes shown in **Fig. 3**. The contact width is decided by the Hertzian contact theory. Assuming the temperature of the sliding contact surfaces of both objects is equal, the distribution ratio of the heat to the pinion can be expressed by the following equation[5]:

$$R_{pp}=\frac{\overline{A_{pp}}}{\overline{A_{pp}}+\dfrac{0.752\,k'_p}{k_p\sqrt{L_{pp}}}} \tag{11}$$

$$\overline{A_{pp}}=\frac{2}{\pi}\left\{\sinh^{-1}\left(\frac{m_{pp}}{l_{pp}}\right)+\left(\frac{m_{pp}}{l_{pp}}\right)\sinh^{-1}\left(\frac{l_{pp}}{m_{pp}}\right)+\frac{1}{3}\left(\frac{m_{pp}}{l_{pp}}\right)^2+\frac{1}{3}\left(\frac{l_{pp}}{m_{pp}}\right)-\frac{1}{3}\left[\left(\frac{l_{pp}}{m_{pp}}\right)+\left(\frac{m_{pp}}{l_{pp}}\right)\right]\sqrt{1+\left(\frac{m_{pp}}{l_{pp}}\right)^2}\right\} \tag{12}$$

Here, k'_p is the thermal conductivity of the pin.

A sliding contact model between the side gear and the pinion is shown in **Fig. 4**. Here, assuming line contact and applying the tooth normal force and relative sliding velocity obtained in the previous section, the theoretical equation for temperature rise θ_{sp} (X_{sp}, Z_{sp}) of the sliding contact surface at the pinion tooth can be expressed by the following equation as well as equation (8):

$$\theta_{sp}\left(X_{sp}, Z_{sp}\right)=\frac{2R_{sp}q_{sp}K_p}{\pi k_p v_{sp}}\int_{X_{sp}-L_{sp}}^{X_{sp}+L_{sp}} e^{-u}K_0\left(\sqrt{Z_{sp}{}^2+u^2}\right)du \tag{13}$$

$$q_{sp}=\frac{\mu F_{np}v_{sp}}{4l_{sp}m_{sp}} \tag{14}$$

$$L_{sp}=\frac{v_{sp}l_{sp}}{2K_p},\quad X_{sp}=\frac{v_{sp}x}{2K_p},\quad Z_{sp}=\frac{v_{sp}z}{2K_p} \tag{15}$$

$$R_{sp}=\frac{\overline{A_{sp}}}{\overline{A_{sp}}+\dfrac{0.752\,k_s}{k_p\sqrt{L_{sp}}}} \tag{16}$$

$$\overline{A_{sp}}=\frac{2}{\pi}\left\{\sinh^{-1}\left(\frac{m_{sp}}{l_{sp}}\right)+\left(\frac{m_{sp}}{l_{sp}}\right)\sinh^{-1}\left(\frac{l_{sp}}{m_{sp}}\right)+\frac{1}{3}\left(\frac{m_{sp}}{l_{sp}}\right)^2+\frac{1}{3}\left(\frac{l_{sp}}{m_{sp}}\right)-\frac{1}{3}\left[\left(\frac{l_{sp}}{m_{sp}}\right)+\left(\frac{m_{sp}}{l_{sp}}\right)\right]\sqrt{1+\left(\frac{m_{sp}}{l_{sp}}\right)^2}\right\} \tag{17}$$

Here, R_{sp} is the distribution ratio of the heat to the pinion, $2l_{sp}$ and $2m_{sp}$ are the contact width and contact length, and k_s is the thermal conductivity of the side gear. For equations (13) through (17), driven and drive surfaces must be considered.

2. 3 Heat Transfer to Oil and Oil Temperature Rise

Estimating the temperature rise of oil which lubricates and cools differential mechanisms is important in the design phase in order to prevent seizure and ensure adequate lubrication. In contrast to the many difficulties involved in directly measuring the temperature of sliding contact surfaces in actual experiments, measuring oil temperature is relatively easy. Likewise, theoretical validation is also easy. Considering this aspect, in this section, an estimation theory for oil temperature rise is examined.

The transfer of heat generated by mixing and sliding contact surfaces is one of the factors which causes oil temperature to rise. In particular, among the general conditions of differential mechanism application, the heat generated by the sliding contact surfaces is considered to significantly raise oil temperature. Here, oil mixing is ignored, and only the heat generated in the sliding contact surfaces is considered as the reason for increased oil temperature.

2. 3. 1 Heat Transfer to Oil

A heat distribution model of sliding contact surface

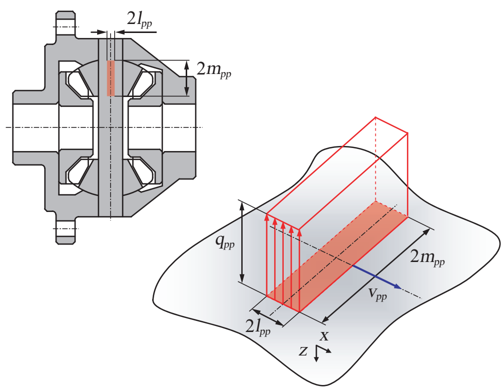

Fig. 3 Sliding contact model between pin and pinion

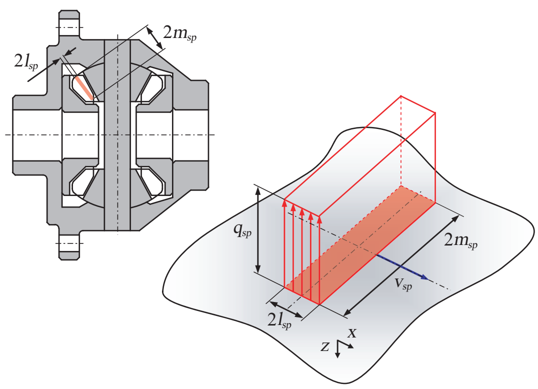

Fig. 4 Sliding contact model between side gear and pinion

between pin and pinion is shown in **Fig. 5**. In the figure, R'_{ppo} and R_{ppo} indicate the heat distribution ratio from the pin and pinion to the oil. In this report, the heat distribution is assumed as the below.

① Frictional heat generated in the sliding contact surface is distributed to the pin and pinion.

② The heat distributed to the respective parts is distributed to the pin and oil and pinion and oil.

Here, assuming the heat distribution ratio to the oil from the pin and pinion are equal, the heat distribution ratio to the oil for frictional heat generated in the sliding contact surface can be expressed by the following equation:

$$R_{ppo} = \frac{h_{pp}\overline{\theta}_{pp}}{R_{pp}q_{pp}} \tag{18}$$

Here, h_{pp} is the convective heat transfer coefficient of the oil. By using equation (8), the average temperature rise on the sliding contact surface of the pinion $\overline{\theta}_{pp}$ is expressed by the following equation:

$$\overline{\theta}_{pp} = \frac{1}{2}\int_{-1}^{1}\theta_{pp}(X_{pp},\ 0)\,dX_{pp} \tag{19}$$

Considering the shape of the sliding contact surface, the transfer of heat from the sliding contact surface to the oil Q_{ppo} can be expressed by the following equation:

$$Q_{ppo} = \frac{4l_{pp}m_{pp}h_{pp}\overline{\theta}_{pp}}{R_{pp}} \tag{20}$$

Furthermore, considering pinion rotation, it can be assumed that the temperature of non-sliding contact surface of the pinion also rise, as illustrated in **Fig. 5**. Considering the transfer of heat from non-sliding contact surface to the oil, the amount of heat can be expressed in the following equation:

$$Q'_{ppo} = 4(\pi r_{pp} - l_{pp})\,m_{pp}h_{pp}\overline{\theta'}_{pp} \tag{21}$$

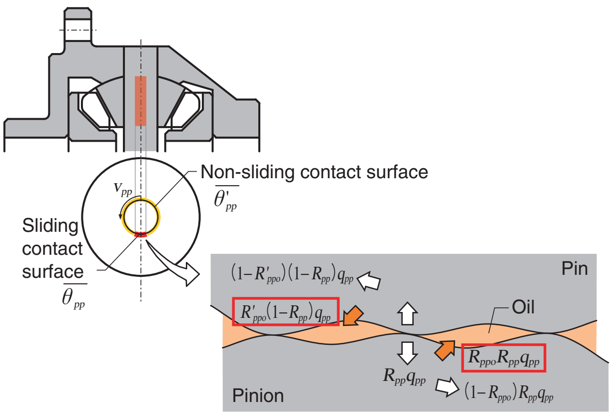

Fig. 5 Heat distribution model of sliding contact surface between pin and pinion

By using equation (8), the average temperature rise of non-sliding contact surface $\overline{\theta'}_{pp}$ is expressed in the following equation:

$$\overline{\theta'}_{pp} = \frac{1}{2}\int_{1}^{\frac{2\pi r_{pp}}{l_{pp}}-1}\theta_{pp}(X_{pp},\ 0)\,dX_{pp} \tag{22}$$

Considering the heat of sliding contact surfaces as a result of side gear and pinion engagement, the transfer of heat from the main sliding contact surfaces to the oil in differential mechanisms Q_o can be expressed by the following equation:

$$Q_o = Q_{ppo} + Q'_{ppo} + Q_{spo} + Q'_{spo} \tag{23}$$

Here, Q_{spo} and Q'_{spo} are the amounts of heat transferred from the sliding contact surface and non-sliding contact surface to the oil as a result of side gear and pinion engagement.

2. 3. 2 Oil Temperature Rise

An extremely simplified heat transfer model of oil is shown in **Fig. 6**. In this paragraph, a theoretical equation for the estimation of oil temperature rise is derived by assuming that the oil receives heat generated from sliding contact surfaces and loses heat into the atmosphere. Considering oil heat transfer, basic equation can be expressed as follows:

$$Q_o dt = \rho_o C_o V_o d\theta + h_a(\theta_o(t) - \theta_a)S_o dt \tag{24}$$

Here, ρ_o and C_o are the oil density and specific heat, V_o and S_o are the oil volume and surface area, h_a is the convective heat transfer coefficient between the oil surface and atmosphere, θ_o is oil temperature, θ_a is atmospheric temperature and t is time. Assuming the oil temperature and atmospheric temperature are equal in the initial state, and equation (24) is solved, oil temperature $\theta'_o(t)$ can be expressed by the following equation:

$$\theta'_o(t) = \frac{Q_o}{h_a S_o}\left\{1 - \exp\left[-\frac{h_a S_o}{\rho_o C_o V_o}t\right]\right\} \tag{25}$$

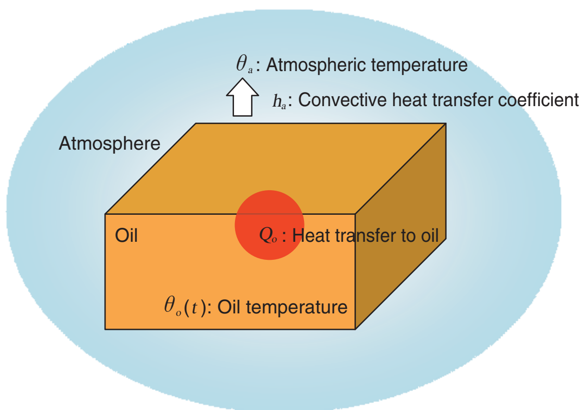

Fig. 6 Heat transfer model of oil

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3. Results and Discussions

In this chapter, the numerical results obtained by applying the theoretical equations for estimating the temperature rise of main sliding contact surfaces and oil are discussed. The validity of these estimation theories is verified by comparing estimated and measured results for oil temperature.

3. 1 Temperature Rise of Sliding Contact Surfaces

The estimation results of temperature rise for the main sliding contact surfaces and non-sliding contact surface are shown in **Fig. 7**, under the calculation conditions shown in **Table 1**. The estimation results in the case that a sliding motion occurs once in the pin and pinion and the side gear and pinion respectively are shown in **Figs. 7** (a) and (b). For both cases, it is estimated that the temperature of the sliding contact surfaces rise sharply, while the temperature of the non-sliding contact surface gradually decline. Also under these conditions, it is estimated that the temperature of the side gear and pinion sliding contact surface rise higher than that of the pin and pinion sliding contact surface.

Table 1 Calculation conditions

Torque	3 200 N·m
Differential rotational speed	20 min^{-1}
Pinion teeth no.	7
Side gear teeth no.	13

Next, estimation results for the maximum and average temperature rises of sliding contact surfaces in relation to torque is shown in **Fig. 8**. The conditions are the same as those shown in **Table 1** excepting torque. The estimation results in the case that a sliding motion occurs once in the pin and pinion and the side gear and pinion respectively are shown in **Figs. 8** (a) and (b). Here, $\theta_{pp,\,max}$ and $\theta_{sp,\,max}$ are the maximum temperature rises of the sliding contact surfaces. From the figures, it is estimated that the maximum and average temperatures of the sliding contact surfaces rise in practically a straight line for both cases as the torque increased. Furthermore, it is estimated that temperature of the side gear and pinion sliding contact surface rise more than that of the pin and pinion sliding contact surface. An issue for the future is how to directly measure the temperatures of these sliding contact surfaces in order to clearly verify the validity of the estimation theory for temperature rise.

3. 2 Oil Temperature Rise

The estimated and measured oil temperatures are shown in **Fig. 9**, under the conditions of **Table 1**. Estimated values are obtained by using equation (25). The figure shows that the estimated and measured values are relatively consistent regarding the oil temperature rise tendencies and converging values. This result proves that the estimation theory for oil temperature rise is valid under these conditions. Moreover, the validity of the estimation theory for the temperature rise of sliding contact surfaces, which is the premise of the estimation

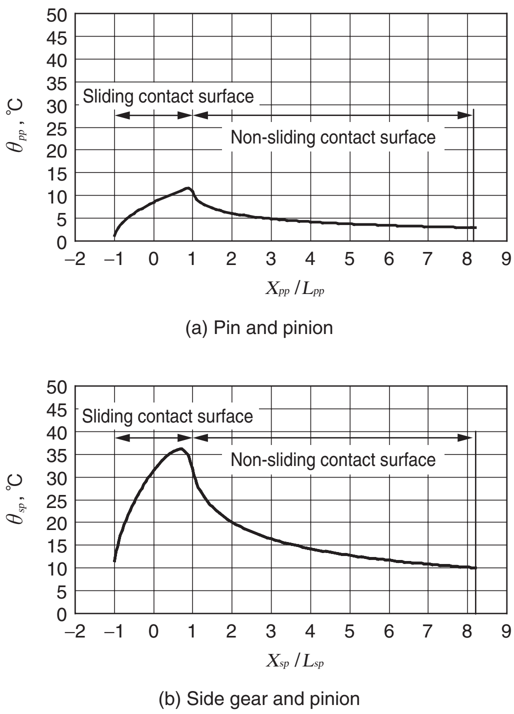

Fig. 7 Estimated results of temperature rise

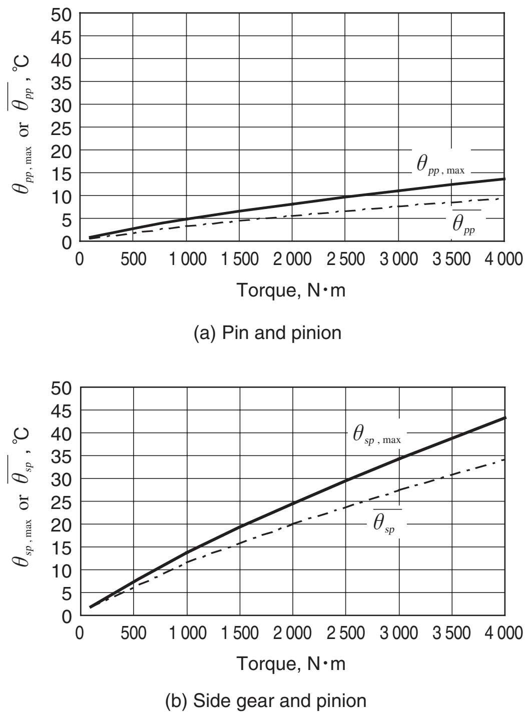

Fig. 8 Estimated results of temperature rise in relation to torque

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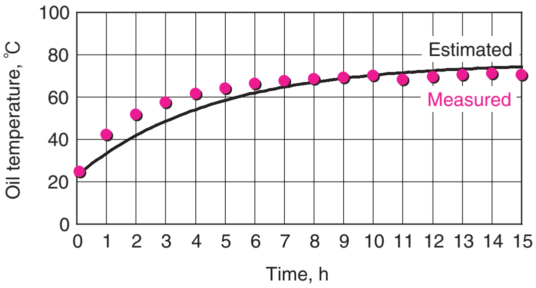

Fig. 9 Comparison result between estimated and measured oil temperature

theory for oil temperature rise, is indirectly verified. In order to improve the accuracy of estimated values, it is necessary to consider the heat capacity of each component and the frictional heat of areas other than the main sliding contact surfaces, as well as examine a heat distribution model.

4. Conclusion

A trial has been carried out to estimate temperature rise on sliding contact surfaces of differential mechanism for automotive drivelines by using the moving heat source theory. Theoretical equations of contact force and relative sliding velocity of main sliding contact surfaces have been derived. Also equations for estimating the temperature rise of main sliding contact surfaces and oil have been derived.

In the numerical examples, the temperature rise of main sliding contact surfaces has been estimated to understand tendency. The validity of this estimation theory has been verified by comparing estimated and measured results of oil temperature.

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