Theoretical Analysis of Flow Characteristics and Bearing Load for Mass-produced External Gear Pump

N. YOSHIDA

This paper presents theoretical equations for calculating pump flow rate and bearing applied load which are important characteristics within external gear pump design. In the external gear pump mass-produced at JTEKT, the eccentricity of the gear housing greatly affects its characteristics. Taking this fact into consideration, I conducted theoretical analysis of leakage through gear tip clearances. This paper describes the equations for the physical model and the analytical solutions of the equations as well as the numerical calculation method for calculating the eccentricity position. It also contains a comparison of the values calculated using my proposed theoretical equations with previous theoretical values and the experimental values. Furthermore, this paper shows examples of calculated results for the theoretical optimum conditions for the eccentricity position.

Key Words: fluid power system, hydraulic pump, external gear pump, leakage flow rate, bearing load, theoretical analysis

1. Introduction

The external gear pump composed of circumscribing twin gears has been used for various machines, such as industrial equipment and automobiles, since ancient times. Its flow rate performance is very high in comparison with other gear type hydraulic pumps, as it can be operated at high pressures over 10 MPa. Therefore it has an advantage in regards to energy saving and device downsizing. JTEKT has been mass-producing the external gear pump driven by an electric motor for the automobile hydraulic power steering system known as H-EPS[®] (Hydraulic-Electric Power Steering), and hopes that it will be used in the future for automotive driveline systems requiring a high pressure hydraulic power source.

In the external gear pump such as that mass-produced by our company which is described in this paper, it is known from experience that the eccentricity of the gear housing greatly influences pump performance and that the delivery flow rate performance increases as the eccentricity increases to a certain extent. The eccentricity state varies due to the following two reasons. One is dispersion of the parts' dimensions caused by machining. The other is the use of journal bearings, which are low in cost. Due to the insufficient supporting rigidity of the journal bearing, the rotating center position of the pump gear varies according to the operating conditions (especially delivery pressure).

Previous studies¹⁾⁻³⁾ took no account of the influence of

the eccentricity of the gears within the detailed theoretical analysis of the flow rate characteristics of the pump, with the exception of Ichikawa's study⁴⁾, which analyzed it theoretically with simple equations. Ichikawa's analysis is straightforward and its equations can be calculated easily. However, it is insufficient in accuracy because the equations used were simple, and mathematical approximations were applied under several assumptions. Additionally, although the eccentricity direction is considered to be the same as the load direction in his analysis, it is not so in our pump with journal bearings. Therefore, previous theoretical analyses are not appropriate for pumps such as our external gear pump.

For the purpose of obtaining theoretical equations to accurately represent the characteristics of the delivery flow rate and bearing load for an actual pump, I have developed a new theoretical analysis of the leakage through the clearances between gear tips with eccentricity and the housing, and the pressure state which is distributed along the gear circumference. This new theoretical analysis is useful in the prediction of the optimization of flow rate performance and its dispersion according to the tolerance of the dimensions, as well as the prediction of bearing load which is very important within the design stage of rotary machines. It also has the ability to elucidate physical mechanisms which significantly influence pump characteristics and contribute to the development of superior new products.

2. Nomenclature

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circumference direction	θ_{z_i}	: angle of contact point <i>i</i> : amount of angle range of gear tip clearance in
	-	circumference direction

 δ : distance between gear center and housing center (eccentricity amount)

Subscripts

- *i* : place number *i* of the gear tip clearance or tooth space within the seal area from the inlet space side
- x : x direction component of force
- y : y direction component of force
- o : central direction component of force

3. Structure of pump





: The transport route of the fluid

Fig. 1 Structure of external gear pump manufactured by JTEKT

The structure of the external gear pump studied in this paper is shown in **Fig. 1**. The gears are assembled in the housing hole, which has an inner diameter slightly larger than the gear addendum diameter. During operation, the drive gear is driven by an electric motor and rotates, and the driven gear rotates by engagement with the drive gear. The gear shaft is supported by a journal bearing. Working fluid (oil) is transported from the inlet space to the outlet space through the tooth spaces of the rotating gear via the route of the side opposite the gear engagement, as shown in **Fig. 1**. It is then pressurized and discharged by the decreasing volumetric change caused by the engagement of the gears.

To understand the actual flow rate of the displacement type pump, it is necessary to take into account not only the theoretical displacement volume $V_{\rm th}$ obtained from the geometric form and the rotational speed $n_{\rm G}$, but also the leakage flow rate. The leakage flow rate refers to the part of the delivered flow rate which flows backward into the inlet space. It is caused by leakage flow from the high-pressure outlet space to the low-pressure inlet space through gaps between the parts of the pump. Taking into account the leakage flow rate $\Delta Q_{\rm P}$, the volumetric efficiency representing the flow rate performance is defined by the following equation⁵.

$$\eta_{\rm V} = 1 - \frac{\Delta Q_{\rm P}}{V_{\rm th} \cdot n_{\rm G}} \tag{1}$$

4. Theoretical analysis

4. 1 Equations for calculating leakage flow rate through gear tip clearances with eccentricity

Figure 2 shows the designated dimensions and definitions of the coordinates (circumference direction position θ and the Cartesian coordinates x and y). Here, the directions of the angle of circumference position and gear rotation are the same, and the positive direction of the leakage flow rate is the opposite direction. Each tooth and tooth space in the sealed area between the gear tips and the housing have been given consecutive numbers starting from the inlet space side shown in **Fig. 2**. In this analysis, the inlet pressure is defined as 0.



Fig. 2 Schematic diagram of gear and housing of pump (Cross section of pump shaft)

When eccentricity of the gear housing arises (eccentricity ratio ε_{G} and eccentricity direction θ_{G}), the gear tip clearance between the gear tip and the housing inner diameter changes according to position angle θ . When the pump operates (i.e. when the gear rotates), during the state in which oil has filled the pump space, the outlet pressure increases relatively by only $\Delta p_{\rm P}$ in respect to the inlet pressure. The gear tip clearance functions to seal oil leakage from the outlet side to the inlet side. In this analysis, the pressure pulsation in the outlet space and the flow unsteadiness are considered to be negligible, and therefore static analysis is conducted with consideration to a steady state. It is assumed that pressure inside each tooth space is uniform, and that there is no inlet/outlet pressure loss at the edge of each gear tip clearance and no change in the viscosity and density of the fluid.

1) Equations for calculating leakage flow rate through single gear tip clearance



Fig. 3 Spatial configuration and fluid condition in the tooth tip clearance

It is assumed that, for the flow in the space of a single gear tip clearance i as shown in **Fig. 3**, the wall surface of the housing can be displayed as a linear plane as the curvatures of the gear and housing are extremely large in respect to the gap amount.

When a sealed tooth exists in the position at the arbitrary angle θ , the gap amount *h* at the gear tip can be approximated by the following equation,

$$h(\theta) = h_0 \{1 - \varepsilon_G \cdot \cos(\theta - \theta_\varepsilon)\}$$
(2)

where the variables are as follows.

$$h_0 = r_{\rm H} - r_{\rm G} \tag{3}$$

$$\varepsilon_{\rm G} = \frac{\dot{o}}{h_0} \tag{4}$$

The position x_i in the flow direction at each gap is expressed by the following equation.

$$\mathbf{x}_{i} = \mathbf{r}_{\mathrm{G}} \left(\boldsymbol{\theta} - \boldsymbol{\theta}_{\mathrm{si}} \right) \Leftrightarrow \boldsymbol{\theta} = \boldsymbol{\theta}_{\mathrm{si}} + \frac{\mathbf{x}_{i}}{\mathbf{r}_{\mathrm{G}}}$$
(5)

The following equations are also introduced.

$$\Delta \theta_{\rm s} = \frac{I_{\rm S}}{r_{\rm G}} \tag{6}$$

$$\theta_{\rm si} = \theta_{\rm sl} + \frac{2\pi(i-1)}{Z_{\rm G}} \tag{7}$$

To consider the influence of the flow velocity in the gap caused by the gear rotation, it is assumed that the gear wall surface moves in the direction shown in **Fig. 3** at the constant velocity U expressed by the following equation.

$$U = 2\pi r_{\rm G} n_{\rm G} \tag{8}$$

In this space, the position x_i in the flow direction is determined from the edge of the upstream side (inlet space side) in the flow passage shown in **Fig. 3**, and the gap amount h_i changes according to x_i . As the pressure is different within each tooth space, the static pressure differential Δp_{Si} exists between the two edges of the flow passage.

In this flow passage space, it could be considered that the flow is laminar as the Reynolds number is very low, due to the fact that the gap amount is much smaller than the flow passage length. Furthermore, it is assumed that the velocity component of the teeth width direction is negligible, since the flow passage width is much larger than the gap amount. Furthermore, it is assumed that the velocity component of the y direction can be overlooked since the change in the gap amount in respect to x is extremely small.

The equation of motion, the boundary conditions and the equation of continuity for the state of pressure pand velocity u of a Newtonian fluid in such a space are expressed by the following simultaneous equations.

$$\frac{dp(\mathbf{x}_{i})}{d\mathbf{x}_{i}} - \mu \frac{d^{2}u(y_{i})}{dy_{i}^{2}} = 0$$
(9)
$$\begin{cases} u(0) = 0, u(h) = U \\ p_{s}(0) = 0, p_{s}(I_{s}) = \Delta p_{si} \end{cases}$$

$$b \int_{0}^{h(\mathbf{x}_{i})} u \, dy = -\Delta Q_{P1}(\mathbf{x}_{i}) = (\text{constant}) \qquad (10)$$

By solving equation (9) by the variable separation method and integrating the solution into equation (10) I obtained the following equation of the leakage flow rate, which depends on the pressure differential Δ_{psi} across both edges of the gear tip clearance h_i .

$$\Delta Q_{\rm P1} = b \left\{ \frac{1}{12\,\mu \int_0^{l_{\rm S}} h_i^{-3} dx} \,\Delta p_{\rm Si} - \frac{U}{2} \, \frac{\int_0^{l_{\rm S}} h_i^{-2} dx}{\int_0^{l_{\rm S}} h_i^{-3} dx} \right\} \tag{11}$$

2) Equation for calculating leakage flow rate in entire pump

The pressure differential $\Delta p_{\rm P}$ across the inlet space and the outlet space equals the total loss of pressure from the several sealing gear tip clearances. Therefore, the following equation was obtained.

$$\sum_{i=1}^{z_{\rm S}} \Delta p_{\rm Si} = \Delta p_{\rm P} \tag{12}$$

By solving equation (11) for pressure differential $\Delta p_{\rm P}$, the following equation was obtained.

$$\Delta p_{\rm Si} = 6\,\mu U \int_0^{l_{\rm S}} h_i^{-2}(x_i) dx + \frac{12\mu \cdot \Delta Q_{\rm P1}}{b} \int_0^{l_{\rm S}} h_i^{-3}(x_i) dx \quad (13)$$

Substituting this in equation (12) and I solved for $\Delta Q_{\rm Pl}$, thus obtaining the following equation of the leakage flow per gear.

$$\Delta Q_{\rm Pl} = \frac{b}{12\mu} \sum_{i=1}^{z_{\rm s}} \left(\int_{0}^{l_{\rm s}} h_i^{-3}(x_i) dx_i \right)^{-1} \\ (\Delta p_{\rm P} - 6\mu U \sum_{i=1}^{z_{\rm s}} \int_{0}^{l_{\rm s}} h_i^{-2}(x_i) dx_i) \\ = \frac{b}{12\mu} \sum_{i=1}^{z_{\rm s}} \left(\int_{\theta_{\rm si}}^{\theta_{\rm si} + \Delta \theta_{\rm s}} h^{-3}(\theta) d\theta \right)^{-1} \\ \left(\Delta p_{\rm P} - 6\mu U \sum_{i=1}^{z_{\rm s}} \int_{\theta_{\rm si}}^{\theta_{\rm si} + \Delta \theta_{\rm s}} h^{-2}(\theta) d\theta \right)$$
(14)

The total leakage flow rate of the pump is expressed as $\Delta Q_{\rm P} = 2 \cdot \Delta Q_{\rm Pl}$, since there are two gears.

4. 2 Calculation of the bearing load force

There are four kinds of pressure acting on the gear circumference, which are divided by their position ranges on the circumference.

• Tooth space closed by the sealing internal diameter of the housing

$$p(\theta) = p_{\mathrm{Mi}}(\theta)$$

$$(\theta_{\mathrm{s}1} + \frac{2\pi}{Z_{\mathrm{G}}}(i-1) + \Delta\theta_{\mathrm{s}} < \theta < \theta_{\mathrm{s}1} + \frac{2\pi}{Z_{\mathrm{G}}}i)$$
(15)

· Gear tip sealed by the internal diameter of the housing

$$p(\theta) = p_{\text{Si}}(\theta)$$

$$(\theta_{\text{sl}} + \frac{2\pi}{z_{\text{G}}}(i-1) < \theta < \theta_{\text{sl}} + \frac{2\pi}{z_{\text{G}}}(i-1) + \Delta\theta_{\text{s}})$$
(16)

· Outlet space in the pump

$$p(\theta) = \Delta p_{\rm P} \qquad (-\theta_{\rm sl} < \theta < -\theta_{\rm Z2}) \tag{17}$$

• Range of closing space formed by the engagement of two gears

$$p(\theta) = \begin{cases} p_{C1} & (\theta_{Z0} < \theta < \theta_{Z1}) \\ p_{C2} & (\theta_{Z2} < \theta < \theta_{Z0}) \end{cases}$$
(18)

Next, I described the derivations of the above mentioned equations (15) to (18)

1) Equations for calculating pressure distribution in sealed areas

I obtained the following equation on the pressure p_{Mi} in each tooth space *i* that is closed by the sealed inner circumference of the housing.

$$p_{\rm Mi} = \sum_{i=1}^{l} \Delta p_{\rm Si} \tag{19}$$

From this, the following equation is obtained by substituting equation (13).

$$p_{\mathrm{Mi}} = 6\mu \left\{ U_{i=1}^{\sum_{j=1}^{i}} \int_{0}^{b} h_{i}^{-2}(\mathbf{x}_{i}) d\mathbf{x}_{i} - \frac{2 \cdot \Delta Q_{\mathrm{P1}}}{b} \sum_{i=1}^{i} \int_{0}^{b} h_{i}^{-3}(\mathbf{x}_{i}) d\mathbf{x}_{i} \right\}$$

$$= 6\mu \left\{ U_{i=1}^{\sum_{j=1}^{i}} \int_{\theta_{\mathrm{Si}}}^{\theta_{\mathrm{Si}} + \Delta \theta_{\mathrm{S}}} h^{-2}(\theta) d\theta - \frac{2 \cdot \Delta Q_{\mathrm{P1}}}{b} \sum_{i=1}^{i} \int_{\theta_{\mathrm{Si}}}^{\theta_{\mathrm{Si}} + \Delta \theta_{\mathrm{S}}} h^{-3}(\theta) d\theta \right\}$$
(20)

Pressure distribution in the gear tip clearance is affected by a phenomenon called the "edge effect". As a result, pressure in the flow passage becomes to higher than that of both edges due to the inflow of oil caused by the movement of the wall in the flow passage direction. Consequently, the pressure in the gap does not simply drop linearly in the flow direction. The following equation obtained from the solution of equations (9) and (10) expresses the pressure distribution caused by the phenomenon.

$$p_{\mathrm{Si}}(\mathbf{x}_{i}) = p_{\mathrm{M}(i-1)} + 12\mu \left(\frac{U}{2} \int_{0}^{x} \frac{1}{h_{i}^{2}} d\mathbf{x}_{i} - \Delta Q_{\mathrm{P1}} \int_{0}^{x} \frac{1}{h_{i}^{3}} d\mathbf{x}_{i}\right)$$

$$\Leftrightarrow p_{\mathrm{Si}}(\theta) = p_{\mathrm{M}(i-1)} + 12\mu \left(\frac{U}{2} \int_{\theta_{\mathrm{Si}}}^{\theta} \frac{1}{h^{2}(\theta)} d\theta - \Delta Q_{\mathrm{P1}} \int_{\theta_{\mathrm{Si}}}^{\theta} \frac{1}{h^{3}(\theta)} d\theta\right)$$
(21)

Analytical solution of the integral calculus shown in the following equations should be applied to the two kinds of definite integrals in equations (14), (20) and (21) obtained above^{60,7)},</sup>

$$\int_{b}^{a} \frac{d\theta}{(1+\epsilon_{\rm G}\cos\theta)^{2}} = \frac{1}{1-\epsilon_{\rm G}^{2}} \left[J(\theta) - \frac{\epsilon_{\rm G}\sin\theta}{1+\epsilon_{\rm G}\cos\theta} \right]_{b}^{a} \tag{22}$$

$$\int_{b}^{a} \frac{d\theta}{(1+\epsilon_{\rm G}\cos\theta)^{3}} = \frac{1}{2(1-\epsilon_{\rm G}^{2})^{2}} \left[(2+\epsilon_{\rm G}^{2})J(\theta) - \frac{3\epsilon_{\rm G}\sin\theta}{1+\epsilon_{\rm G}\cos\theta} - \frac{(1-\epsilon_{\rm G}^{2})\epsilon_{\rm G}\sin\theta}{(1+\epsilon_{\rm G}\cos\theta)^{2}} \right]_{b}^{a} \tag{23}$$

$$J(\theta) = \frac{2}{\sqrt{1-\epsilon_{\rm G}^{2}}} \tan^{-1} \left[\sqrt{\frac{1-\epsilon_{\rm G}}{1+\epsilon_{\rm G}}} \frac{\sin\theta}{1+\cos\theta} \right] \tag{24}$$

where the domain of function (24) is $-\pi/2 \le \theta \le \pi/2$, as it is not continuous at $\theta = \pi/2$. Furthermore, both *a* and *b* of equations (22) and (23) are constants.

2) Other forces acting on gear circumference

The forces acting on the gear circumference excluding the distributed pressure in the area sealed by the housing are as follows: the force due to the pressure $\Delta p_{\rm P}$ in the outlet space, the closing pressure $p_{\rm Ci}$, and the engagement force $F_{\rm Zi}$.

Two closed spaces are formed by the gear

circumference around the two contact points of the gears. The volume of these closed spaces changes due to gear rotation, however the pressure inside can become to be higher than that of the pump outlet. This is due to the difficult compressibility of the fluid, and because a flow passage with insufficient area forms between the two tooth surfaces or in a notch of the side plate. This paper omits description of this theoretical analysis.

When two gears engage each other with contact at one or two points as shown in **Fig. 2**, the engagement force acts in the circumference direction (θ) and its magnitude is expressed in the following equation, where the torque $T_{\rm Pi}$ is the torque received at each engagement point *i*.

$$|F_{\rm Zi}| = \frac{T_{\rm Pi}}{r_{\rm Zi}}, (i = 1, 2)$$
 (25)

$$T_{\rm P} = T_{\rm P1} + T_{\rm P2}$$
 (26)

The actual pump torque $T_{\rm p}$ is expressed by the following equation, in which several losses are not taken into account because the dominant factor is the act of high-pressure pumping.

$$T_{\rm P} = \frac{V_{\rm th}}{2\pi} \Delta p_{\rm p} \tag{27}$$

The central direction component of the engagement force F_{Zi} is expressed by the following equation, where α_{G} is the engagement pressure angle of the gear.

$$|F_{\rm Zio}| = |F_{\rm Zi}|\sin\alpha_{\rm G} \tag{28}$$

The following equations, which present the component of the x direction and that of the y direction in the Cartesian coordinates of the central direction component of the engagement forces, are derived from equations (25) to (28).

$$|F_{Zx}| = -\sum |F_{Zio}| \cos \theta_{Zi}$$

$$= -\frac{V_{th}}{2\pi} \Delta p_{p} \sin \alpha_{G} \sum_{i=1}^{2} \frac{\cos \theta_{Zi}}{r_{Zi}}$$

$$|F_{Zy}| = -\sum |F_{Zio}| \sin \theta_{Zi}$$

$$= -\frac{V_{th}}{2\pi} \Delta p_{p} \sin \alpha_{G} \sum_{i=1}^{2} \frac{\sin \theta_{Zi}}{r_{Zi}}$$
(30)

3) Equations for calculating force of bearing load

The force applied to the bearing is the central direction component of the force acting on the gear circumference and can be calculated using the pressure distribution $p(\theta)$ expressed in equations (15) to (18) and the engagement forces expressed in (29) and (30). The x direction component, the y direction component and the magnitude of these forces are expressed in the following equations.



$$|F_{\rm Gx}| = -r_{\rm G}b \int_0^{2\pi} p(\theta)\cos\theta \,d\theta + |F_{\rm Zx}| \tag{31}$$

$$|F_{\rm Gy}| = -r_{\rm G}b \int_0^{2\pi} p(\theta)\sin\theta \,d\theta + |F_{\rm Zy}| \tag{32}$$

$$|F_{\rm G}| = \sqrt{|F_{\rm Gx}|^2 + |F_{\rm Gy}|^2} \tag{33}$$

4. 3 Method for calculating pump characteristics with consideration to gear support by journal bearings

From the aforementioned equations, the pump characteristics can be calculated when the rotational center position of the gear is known. However, when using journal bearings, the position of the gear rotational center is not known, and must therefore be calculated. This is because both the bearing load force vector (which depends on pressure distribution) and the rotational center position of the gear (which depends on the bearing load force vector) are dependent on each other.

It is therefore necessary to conduct numerical iteration calculation to solve the equations, under the condition that the bearing load force vector and journal bearing supporting force vector are equal. **Figure 4** shows the flow chart of the calculation program.



Fig. 4 Flow chart for calculating characteristics of journal bearing support structure pump

It is obvious that the bearing characteristics of the external gear pump obey the previous established theory. In this paper, the bearing performance numerical values used in the calculation are referenced from a separate document⁷⁾.

5. Results of calculations

Table 1 Geometry of actual gear pump (Typical values)				
Radius of gear addendum : $r_{\rm G}$, mm				
Number of teeth on gear : z_{G}				
Number of sealed teeth on gear : z_s				
Gear tip circumference length : l_{s} , mm				
Face width of gear : b, mm				
Theoretical pump displacement per revolution : $V_{\rm th}$, cc/rev.				
Rotational speed of gear : $n_{\rm G}$, min ⁻¹				
Fluid viscosity : μ , Pa·s				
Pressure differential across pump : $\Delta p_{\rm P}$, MPa				

Table 1 Geometry of actual gear pump (Typical values)

5.1 Comparison with values from previous theory

A comparison between the calculated values of the volumetric efficiency according to the proposed theory and those of the previous theory⁴ is shown in **Fig. 5**. The calculation conditions are the dimensions and operation conditions described in **Table 1**.

As shown in **Fig. 5**, the values between the previous theory and the proposed theory have very little difference only when the eccentricity ratio is small. Also, the value of volumetric efficiency of the proposed theory differs greatly from that of the previous theory when the nonlinearity of its change in the proposed theory, in regard to the eccentricity ratio, is large. An eccentricity ratio equal to 1 physically contradicts a volumetric efficiency that is not equal to 1, since this refers to the occurrence of leakage.

In the derivation of the equations in the previous theory, the fundamental equations are based on a theoretical equation based on constant gap amount, and higher order components in the equations are omitted in order to easily obtain solutions. Therefore, as shown in **Fig. 5**, approximate accuracy is poor and there are large errors when the conditions are not limited to a lower eccentricity ratio, or to a small nonlinearity of the change in volumetric efficiency in regard to the eccentricity ratio.

As a result, the proposed theoretical equations have much higher accuracy than the previous equations in the case of a large eccentricity ratio. The bearing load characteristic is the same as well, since its calculation equations depend on the leakage flow rate.



Fig. 5 Comparison of proposed theoretical values and conventional theoretical values

5. 2 Comparison between theoretical values and experimental values

Figure 6 shows the results of a comparison between experimental values from an actual pump and the theoretical values. Typical pump conditions are described in **Table 1**.

The experimental values shown in **Fig. 6** (a) are measured under the condition where the pump speed is constant and the pump outlet pressure has been changed by altering the outlet side pipe resistance. The two tested pumps differ only in the dimensions which influence the amount of gear tip clearances. The values show that the calculation results of the pump flow rate using the proposed theoretical equations are larger than the experimental one. This difference is derived from the fact that the leakage occurs through gaps other than that of the gear tip on the actual pump. Inferring that leakage through other gaps with constant clearance besides the gear tip clearance is directly proportional to the pressure, it would be physically justifiable that this difference increases with an increase in pressure.

To validate the accuracy of the theoretical value of only the leakage through gear tip clearances, **Fig. 6 (b)** shows the investigated results obtained by changing the y direction position of the center of the journal bearing shown in **Fig. 2** so that only the gear tip clearances change. On the axis of ordinate, the change in leakage flow rate is plotted against the leakage flow rate when the abscissa is at zero. The leakage flow rate is calculated through equation (1) after solving for $\Delta Q_{\rm p}$ and substituting the pump flow rate $Q_{\rm p}$ measured in the actual machine. According to this graph, it is evident that the theoretical value nearly coincides with the experimental value. This proves that the proposed theoretical equations can predict loss due to the leakage of the actual pump flow rate from the gear tip clearances.



(b) Amount of change due to component dimension affecting tooth tip clearance

Fig. 6 Comparison of experimental values and theoretical values of pump flow rate

5. 3 Discussion of theoretical optimum conditions

Figure 7 shows the theoretical calculation results of the pump flow rate and the bearing load obtained by changing the eccentricity direction and the eccentricity ratio. The pump flow rate characteristic is expressed as the volumetric efficiency, and bearing load is expressed as the normalized amount when the value of the eccentricity ratio is zero. The reason that the volumetric efficiency may be over 1 is that fluid oil is dragged and transported by the gear tip outer surface due to the viscosity of the oil.

These results indicate the following facts: the characteristics of the flow rate are influenced greatly by the eccentricity ratio, but not so much by the eccentricity direction, and the eccentricity direction is optimum at 180 degrees when the eccentricity ratio is small.

It is obvious that the bearing load decreases when eccentricity arises in the direction of the inlet space side. On the other hand, the bearing load increases significantly when eccentricity arises in the direction around 180 degrees. This is because, when sealing is conducted on the inlet space side, the y direction components of the forces acting due to the pressure distributed along the circumference of the gear cancel each other out in the second and third quadrants of the coordinate space in **Fig. 2**, and the x direction components counteract the closing pressure and engagement force and decrease. In addition, the bearing load increases with an increase in the eccentricity ratio because the pressure is distributed unevenly. Taking into account that the life of the rolling element type bearing is generally inversely proportional to approximately the cube of the load⁸, attention must be given during design because even a change in the load as small as that in **Fig. 7** (**b**) greatly influences the reliability of the product.



(b) Bearing applied load characteristics

Fig. 7 Eccentricity ratio of theoretical value and changes in eccentric direction

6. Conclusion

I have proposed new theoretical equations for the external gear pump to calculate the bearing load and the leakage flow rate caused by gear tip clearances. In addition, I have created a program that calculates the numerical values of the unknown eccentricity position of the gear when the bearing type of the pump is a journal bearing. By investigating the proposed theoretical equations, the following conclusions were drawn.

- This theory takes into account the eccentricity of the gear more correctly than the previous theory, and its calculation results are much different from those of the previous theory, except for the local conditions.
- When the experimental values are compared with the calculated values according to the proposed theory, the calculated values are confirmed as valid.
- The leakage flow rate through the gear tip clearances greatly changes according to the eccentricity ratio.
- The bearing load changes greatly due to the eccentricity direction.

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* Hydraulic System Engineering Dept., Driveline Systems Operations Headquarters