

# Model-Free Control of Longitudinal and Lateral Dynamics for Automated Vehicles



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*The development of “Advance Driver Assistance Systems” (ADAS) will intensify in the future and contribute to the deployment of Intelligent Transportation Systems (ITS), thus improving the safety and fluidity of traffic, as well as energy consumption. For that purpose, a new “Model-Free Control” (MFC) approach is presented in the context of longitudinal and lateral motion of a car, leading to “intelligent controllers”, easy to implement. Simulation results, obtained on realistic environments, have shown the efficiency of our approach.*

**Key Words:** Automotive control, Longitudinal and lateral control, model-free control, algebraic estimation, ADAS (Advanced Driving Assistance Systems)

## 1. Introduction

Our planet is confronted today with 4 major problems:

- Energy: In less than a century, man has almost exhausted the fossil resources of the planet.
- Pollution: 1 billion cars are running, 60% of which are in North-America, Europe and Asia (i.e. 15% of the world population). These cars produce greenhouse gases with the consequences we know about global warming.
- Security: According to WHO data for 2015, we deplore 1.3 million deaths per year (10% in rich countries) due to road traffic accident. Moreover, 95% of accidents are due to driver failures.
- Congestion: There is an imbalance between supply and demand for driving surface, hence the need to improve and regulate the use of roads.

The interest is therefore clear for the development of new technologies for autonomous cars, or more generally “intelligent transport system” (ITS)<sup>18)</sup>.

In Europe, the Eureka PROMETHEUS program (PROgram for a European Traffic of High Efficiency and Unprecedented Safety) began in 1985 and was completed in 1994, the date of the first ITS congress in Paris. This international congress is now organized every year in various countries of the world, and many such ITS programs have been conducted at the European level, but also in the USA (PATH program in California), Japan, Korea and now in China, where more than 20 million cars have been produced in 2014.

Moreover, the development of new localization (Global Positioning System) and navigation technologies

(Geographic Information System and Digital Mapping System) as well as proprioceptive sensors (accelerometers, gyrometers, odometers, etc.) and exteroceptive sensors (embedded cameras, radars, lasers, etc.) have made the implementation of autonomous vehicles and intelligent transport systems possible.

Therefore, car manufacturers have developed a range of ADAS (Advance Driver Assistance System) to assist the drivers, and subsequently improve safety, comfort, efficiency and energy consumption. Many studies have shown the positive impact of ADAS on traffic accidents and fuel consumption<sup>13)</sup>.

Among them, one can cite the airbag and the seat belt which are “passive” and also the ABS (Antilock Brake System) and the ESP (Electronic Stability Program) “active”, which are preventing many crashes<sup>16)</sup> and which are now mandatory in all new vehicles<sup>14)</sup>. Let mention other optional passive and active ADAS:

- Obstacle and collision warning system: Detecting obstacles and warning the driver about an imminent collision (passive).
- Lane-keeping system: Warning the driver when the vehicle leaves the lane unintentionally (passive).
- Emergency braking system: Detecting obstacles and notifying the driver about an imminent collision (passive). If the collision is deemed unavoidable, the system will automatically brake (active, depending on the driving situation).
- Eco driving support system: Providing the driver with optimal set point velocity and suitable gear selection to keep driving in a more environmental friendly manner, reducing fuel consumption (passive).

- Longitudinal control system: Using exteroceptive and proprioceptive sensors to regulate the car velocity and the inter-distance with the front vehicle, through control of the acceleration or the braking pedal (active).
- Lateral control system: Using exteroceptive and proprioceptive sensors to maintain the vehicle on the lane, or to produce a lane change to safely overtake the front vehicle, through the control of the steering wheel (active).

In this paper, we will focus on the last two ADAS, since these longitudinal and lateral controls improve safety, traffic flow, comfort, consumption and reduce safety distances, thus increasing the flow area and the traffic flow. The vehicle longitudinal and lateral control problem has been widely investigated in the literature via model-based techniques<sup>(1), 2), 4), 6), 12), 17), 20), 23), 24)</sup>. It is obvious that such approaches depend on the calibration of the corresponding models. Indeed, their performances are guaranteed whereas the model is valid, which is difficult to guarantee. This is the reason why we have progressively developed a model-free approach, leading to control algorithms easily implementable in an embedded context.

In **Section 2**, we recall the evolution of our developments on longitudinal and lateral vehicle control problem, first based on a nonlinear 3DoF (Degrees of Freedom) model. In **Section 3**, we present a short summary on the model-free control approach and then we apply it in **Section 4** to the design of a longitudinal/lateral vehicle control. Finally, we give some simulation results and words of conclusion in **Section 5**.

## 2. Previous Developments on Model-based Longitudinal and Lateral Control

In the case of trajectory tracking implying coupled longitudinal and lateral dynamics, we have considered the nonlinear 3DoF bicycle model, which is briefly recalled in equation (1):

$$\begin{aligned}
 m (\dot{V}_x - \dot{\psi}V_y) &= F_{xf} + F_{xr} \\
 m (\dot{V}_y + \dot{\psi}V_x) &= F_{yf} + F_{yr} \\
 I_z \ddot{\psi} &= L_f F_{yf} - L_r F_{yr}
 \end{aligned} \tag{1}$$

where:

- $m$  denotes the vehicle mass,
- $V_x$  the longitudinal velocity of the car,  $V_y$  its lateral velocity,
- $\dot{\psi}$  the yaw rate,
- $I_z$  the yaw moment of inertia,
- $F_{xf}$  and  $F_{xr}$  the front and rear longitudinal tire forces,
- $F_{yf}$  and  $F_{yr}$  the front and rear lateral tire forces,
- $L_f$  and  $L_r$  the distances from the center of gravity of the car to the front and rear axles.

The front and rear longitudinal forces can be expressed using the rotation dynamics of the wheels:

$$\begin{aligned}
 F_{xf} &= \frac{1}{R} (-I_r \dot{\omega}_f + T_e - T_{bf}) \\
 F_{xr} &= \frac{1}{R} (-I_r \dot{\omega}_r - T_{br})
 \end{aligned} \tag{2}$$

$R$  denoting the wheel radius,  $I_r$  the wheel inertia,  $\omega_f$  and  $\omega_r$  the rotation velocity of the front and rear wheel,  $T_e$  the engine torque providing the car acceleration,  $T_{bf}$  and  $T_{br}$  being the braking front and rear torques.

Under the assumption of small slip angles, linear lateral tire forces models can be considered as follows, with front and rear cornering stiffness coefficients  $C_f$  and  $C_r$  and steering angle  $\delta$ :

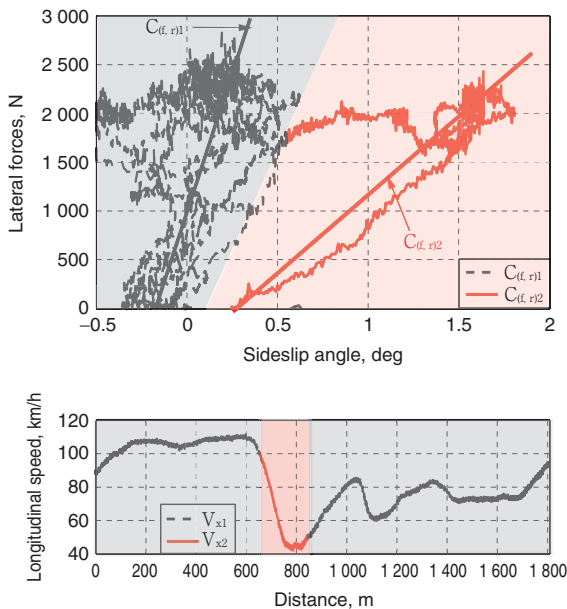
$$\begin{aligned}
 F_{yf} &= C_f \left[ \delta - \frac{V_y + \dot{\psi}L_f}{V_x} \right] \\
 F_{yr} &= -C_r \left[ \frac{V_y - \dot{\psi}L_r}{V_x} \right]
 \end{aligned} \tag{3}$$

The vehicle model (1)-(3) has been shown to be “differentially flat”<sup>\*1</sup>. This property allowed us to design nonlinear controllers making the tracking error exponentially converge to zero<sup>(9), 20)</sup>.

Unfortunately, this control approach has been shown to be not sufficiently robust to parameter uncertainties such as for example tire road forces which are highly nonlinear and difficult to estimate, as shown in **Fig. 1**, which illustrates tire characteristics during an experimental braking maneuver on a real race track. This maneuver has pointed out the different nonlinear dynamics of the tire forces which cannot be well represented in that driving situation by linear tire models, and the cornering stiffness coefficients are clearly dependent on the longitudinal velocity of the car. Moreover, other critical driving situations exist, for which simplified vehicle models cannot provide a realistic behavior of the actual car to design efficient model-based controllers, such as the rollover phenomenon due to high values of load transfer ratio (LTR), under-steering or over-steering when braking in a turn due to high values of the front and rear sideslip angles. For these reasons, we have then developed a vehicle controller based on a “model-free design”<sup>(7)</sup> but using flat outputs<sup>(3), 19)</sup>. Despite that the corresponding numerical results were good, this solution was suffering from the fact that one of the two flat outputs was depending on some uncertain physical parameters,

\*1 Flatness is a system property which characterizes controllability of a nonlinear system. In a flat system, the inputs and states can be computed from the output and a finite number of its derivatives.

rather difficult to identify, such as e.g. the vehicle mass. This implies that an accurate tracking trajectory is not guaranteed, especially when the trajectories are characterized by tight bends which involve high dynamical loads. So, obtaining a “good mathematical model” in general driving situations is a really difficult task, if not an impossible one. Consequently, we decided to apply a “model-free approach” using two “natural outputs” which can be easily computed from onboard measurements, these outputs being the longitudinal velocity and the lateral deviation of the car. Let first recall the general “model-free control approach” before presenting the design of the resulting “intelligent controllers” applied in the context of longitudinal and lateral control of wheeled vehicles.



**Fig. 1** Nonlinear behavior of experimental lateral tire force characteristics

### 3. A Background of the Model-Free Control Approach

Model-free control was already applied and used quite successfully in various concrete examples<sup>(7), (8), (10), (15)</sup>. For obvious reasons, let insist here on its applications to intelligent transportation systems<sup>(5), (20), (24)</sup>.

The input-output relation can be expressed in a small time-interval as follows, where  $z$  is the output and  $u$  the input variable:

$$z^{(\nu)} = F + \alpha u \tag{4}$$

where  $\alpha$  and  $\nu$  are constants, chosen by the practitioner, so that:

- $\nu$  is the time-derivation order,
- $\alpha u$  and  $F$  have the same order of magnitude.

Until now from our knowledge, in the context of model-free control, in all the existing concrete examples  $\nu = 1$  or  $2$ .

Some comments and assumptions on  $F$  can be done:

- $F$  is estimated via the measurements of the control input  $u$  and the controlled output  $z$ .
- $F$  does not distinguish between the unknown model of the system and the perturbations and uncertainties.

Let  $\nu = 2$  in equation (4). It can be rewritten:

$$\ddot{z} = F + \alpha u \tag{5}$$

The corresponding intelligent-Proportional-Integral-Derivative (i-PID) controller, reads:

$$u = -\frac{F - \ddot{z}^d + K_p e + K_i \int e dt + K_d \dot{e}}{\alpha} \tag{6}$$

where  $z^d$  is the desired signal,  $e = z - z^d$  the tracking error and  $K_p$ ,  $K_d$  and  $K_i$  are the usual PID (Proportional Integral Derivative) gains. In fact,  $u$  given by (6) is said an i-PID controller since the term  $F$  is compensated. In fact, combining equations (5) and (6) leads to:

$$\ddot{e} + K_p e + K_i \int e dt + K_d \dot{e} = 0 \tag{7}$$

If  $K_i = 0$ , we obtain the intelligent-Proportional-Derivative (i-PD) controller.

Now, if we consider  $\nu = 1$  in equation (4) we have:

$$\dot{z} = F + \alpha u \tag{8}$$

and setting  $K_i = 0$ , we obtain the intelligent-Proportional (i-P) controller, which turns out until now, to be the most useful intelligent controller:

$$u = -\frac{F - \dot{z}^d + K_p e}{\alpha} \tag{9}$$

and combining equations (8) and (9) leads to:

$$K_p e + \dot{e} = 0 \tag{10}$$

meaning that, as in equation (7), the tracking error exponentially converges to 0. But, it should be pointed out that the term  $F$  in the expression of the i-PD controller (7) or i-P controller (9) should be well known to be compensated. But as we have mentioned before, this term  $F$  usually corresponds to un-modeled dynamics, perturbations or uncertainties which are not known. Therefore,  $F$  has to be estimated. According to the algebraic parameter identification developed in (10), (11), where the probabilistic properties of the corrupting noises may be ignored,  $F$  will be estimated on small time intervals  $[t-\tau, t]$  by a piecewise constant function  $F_{est}$ . For that purpose, let us rewrite equation (8) in the operational domain, where  $Z$ ,  $U$  and  $\frac{\Phi}{s}$  denote respectively the Laplace transform of  $z$ ,  $u$  and  $F$  which is supposed to be

constant on  $[t-\tau, t]^{25}$  :

$$sZ = \frac{\Phi}{s} + \alpha U + z(0) \quad (11)$$

We get rid of the initial condition  $z(0)$  by left-differentiating both sides of equation (11) with respect to  $s$ , which gives:

$$Z + s \frac{dZ}{ds} = -\frac{\Phi}{s^2} + \alpha \frac{dU}{ds}$$

Then, noise attenuation is achieved by left-multiplying both sides by  $s^{-2}$ , leading to integral expressions in the time domain (using inverse Laplace transform):

$$F_{est}(t) = -\frac{6}{\tau^3} \int ((\tau - 2\sigma)z(\sigma) + \alpha\sigma(\tau - \sigma)u(\sigma)) d\sigma \quad (12)$$

The extension to the case  $v = 2$  is straightforward, and the resulting estimation of  $F$  is given by the following expression<sup>25)</sup> :

$$F_{est}(t) = -\frac{60}{\tau^5} \int (\tau^2 + 6\sigma^2 - 6\tau\sigma)z(\sigma) d\sigma - \frac{30\alpha}{\tau^5} \int (\tau - \sigma)^2 \sigma^2 u(\sigma) d\sigma \quad (13)$$

Let us now develop this model-free control approach in the context of vehicle control.

#### 4. Longitudinal and Lateral Model-Free Control of the Car

As mentioned in the previous section, an appropriate choice of inputs and outputs is required to take advantage of the model-free control approach. To avoid any modeling problem and ensure a desired tracking for longitudinal and lateral motions, the following input and output variables are selected.

- The first input  $u_1$  is the acceleration or braking torque, and the first output  $y_1$  is the longitudinal velocity  $V_x$  of the car.
- The second input  $u_2$  is the steering wheel angle, and the second output  $y_2$  is the lateral deviation of the car (with respect for example to the center of the road).

It is obvious that the second output, which is the lateral deviation of the car, can be expressed from  $V_x$ ,  $V_y$  and the yaw rate  $\dot{\psi}$ . This allows to include some coupling effects between the longitudinal and lateral motions. According to the background on the model-free setting, the above inputs/outputs, and Newton second law, the following two local models are considered:

$$\text{Longitudinal local model : } \dot{y}_1 = F_1 + \alpha_1 u_1 \quad (14)$$

$$\text{Lateral local model : } \ddot{y}_2 = F_2 + \alpha_2 u_2 \quad (15)$$

Equations (14) and (15) seem to be decoupled, but the coupling longitudinal and lateral effects are included in the terms  $F_1$  and  $F_2$ . Let us also point out that equation (15) is an order 2 formula with respect to the time derivative of  $y_2$ , this is due to Newton second law. So equation (15) will be closed by an i-PD of the form (6) with  $K_i = 0$  and  $e_2 = y_2 - y_2^d$ :

$$u_2 = -\frac{F_2 - \ddot{y}_2^d + K_p^2 e_2 + K_d^2 \dot{e}_2}{\alpha_2} \quad (16)$$

and equation (14) will be closed by an i-P controller of the form (9) with  $e_1 = y_1 - y_1^d$ :

$$u_1 = -\frac{F_1 - \dot{y}_1^d + K_p^1 e_1}{\alpha_1} \quad (17)$$

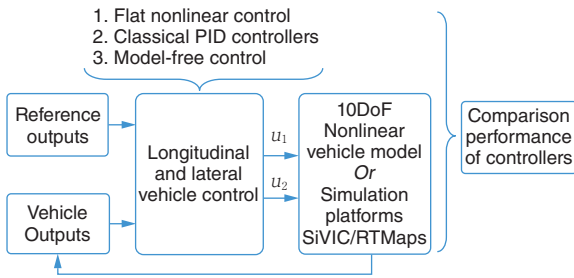
Let us notice that  $F_1$  in equation (17), respectively  $F_2$  in equation (16), will be estimated using equation (12) with  $z = y_1$  and  $u = u_1$ , respectively using equation (13) with  $z = y_2$  and  $u = u_2$ .

**Remark:** In the above development, the inputs have been assumed as ideal actuators (instantaneous response to any command). For example, in a vehicle, the steering wheel angle is controlled and consequently has a finite time response. However, this response is assumed much faster than that of the yaw dynamics of the vehicle. Typically, the angle control of the steering system is developed by the steering supplier. JTEKT has developed an angle control strategy, based on a similar control approach than that presented in this paper, where unknown dynamics is estimated and compensated for ensuring high level of robustness<sup>22)</sup>.

#### 5. Results and Conclusion

Embedding ADAS on cars is known to be a difficult task, from a technical point of view, but also due to the public perception and acceptance of possible interventions as well as in reliability and safety of the system. In this context, simulation is a necessary step for validation before actual vehicle implementation. For that purpose, many platforms have been developed, such as the interconnected pro-SiVIC/RTMaps prototyping platform described for example in 21). Simulation allows to approach the reality of a situation by playing for example with weather factors or studying the influence and the impact of physical parameters on the robustness of the functions used in embedded applications. Difficult or dangerous tests are also easily accessible such as emergency braking due to obstacle detection or road friction problems or road departure warning.

In this study, the simulation stage is carried out according to the block diagram of **Fig. 2**, using a precise 10DoF nonlinear vehicle for simulation model, and actual data, previously recorded on a race track with a



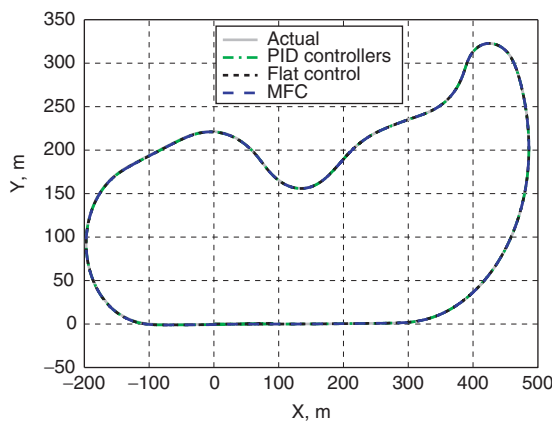
**Fig. 2** Block diagram of vehicle controllers, reference trajectory reconstruction and vehicle models

car prototype, as reference trajectories which should be followed by the car.

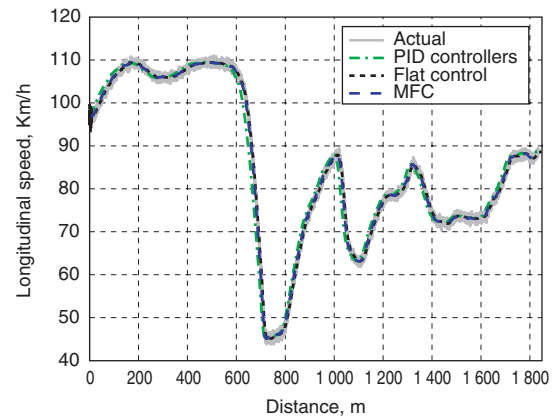
The efficiency and relevance of the model-free control as well as its performances are compared to those of nonlinear flat control and classical PID controllers.

Two simulation scenarios are conducted for two values of the road friction coefficient: the first one for dry asphalt  $\mu = 1$  and the second one for wet asphalt  $\mu = 0.7$ . These scenarios mean that the adhesion capability of the ground is reduced, thus, the vehicle maneuverability and controllability become more difficult.

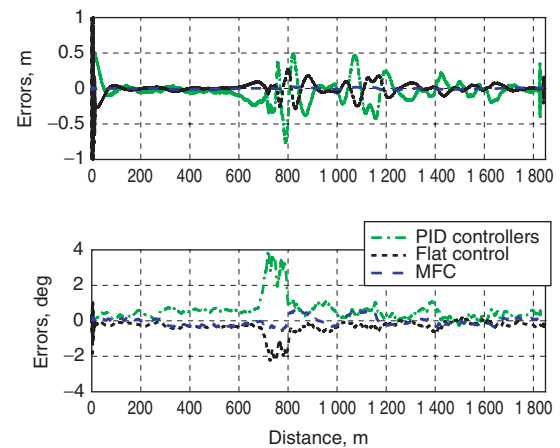
For dry road, results are shown in **Figs. 3, 4, 5** and **6**. These simulations demonstrate that the model-free control (MFC) gives quite satisfying results, especially on **Fig. 5** showing that the tracking errors on the lateral deviation and on the yaw angle outputs produced by the MFC are much better compared to those produced by the other controllers. These errors are less than 10 cm and 0.5 deg in the case of MFC. It should be noticed that the test track which has been considered implies strong lateral and longitudinal dynamical requests. This track involves different types of curvatures linked to straight parts, and all these configurations represent a large set of driving situations. Finally, **Fig. 6** shows that the control signals are quite close to the actual ones provided by the driver



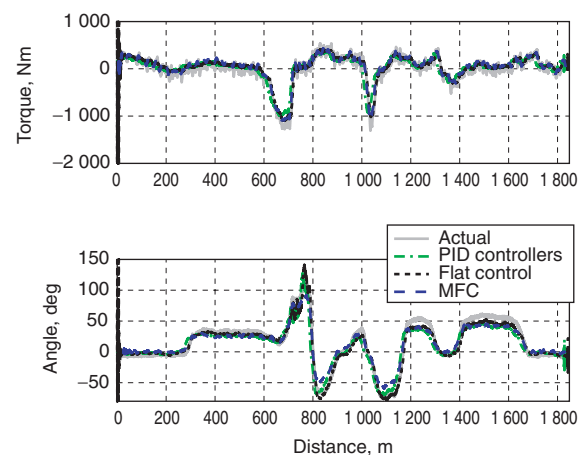
**Fig. 3** The reference trajectory versus the closed-loop simulated trajectories



**Fig. 4** The actual longitudinal speed versus the closed-loop simulated longitudinal velocities



**Fig. 5** Tracking trajectory errors on lateral deviation and yaw angle



**Fig. 6** Wheel torques and steering angles control signals: actual and simulated



along the track race.

Moreover, in the case of wet road, all normalized errors are deteriorated as expected, but using the model-free control, the maximum normalized error values are less than 3.5%, which is much better than with the other controllers.

Consequently, the relevance of this new model-free control approach, even in extreme driving conditions, has been validated. Let us also point out that the resulting control algorithms are easy to implement and that they use measurements which are available on most vehicles.

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