Non-linear Control of Yaw Rate Response in Steer-by-Wire Vehicle*1

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In this study, a method to independently control the yaw rate gain and phase characteristics is proposed. In steer-by-wire vehicles, it is possible to design the characteristics of vehicle dynamics such as the yaw response. However, the gain and phase of the frequency response of linear systems are coupled which limits the tunability. To deal with this issue, this study proposes a control method that decouples the gain and phase characteristics of the yaw rate dynamics by using a non-linear filter based on a NARX (Non-linear Auto Regressive eXogenous) model. The proposed control is validated on a steer-by-wire equipped vehicle. The control strategy is described and experimental data demonstrate improvements to maneuverability and stability.

***Key Words**: SBW (steer-by-wire), yaw rate control, yaw rate response, vehicle dynamics*

1. Introduction

Steer-by-Wire (hereinafter "SBW") steering systems, in which the Human Machine Interface (hereinafter "HMI") unit operated by the driver and the steering unit which controls the road wheel angle are mechanically separate, are expected to improve the degree of vehicle control freedom and mounting flexibility on vehicle[1]. They are highly compatible with Advanced Driver Assistance Systems (hereinafter "ADAS") and automated driving and can prevent interference between commands from the vehicle system and hand wheel operation and collaborated with them[2]. In regards to cabin packaging, these systems improve mounting flexibility of the HMI unit, and the Stowable Steering Column that the hand wheel can be stowed to near the dashboard during automated driving has been proposed[3]. Being able to independently control the hand wheel and road wheel is also expected to reduce workloads and improve maneuverability during manual driving. Variable ratio control adjusting the angle ratio of the road wheel angle and hand wheel angle (hereinafter, "steering ratio") make possible to reduce the amount of hand wheel operation at low vehicles speeds by setting the steering ratio according to the vehicle speed. Dynamic control of the relationship between the hand wheel angle and road wheel angle can change the characteristics of the vehicle in response to hand wheel operation and improve maneuverability and stability. This report proposes a control method to change the frequency response of yaw rate to hand wheel angle (hereinafter "yaw rate response") as an example of this dynamic control.

Previously the focus for yaw rate response was on contributing to stability in high vehicle speed ranges, however it is thought that drastically reducing the steering ratio (hereinafter "low ratio") in low vehicle speed ranges may also affect maneuverability. This report will show the effects on yaw rate response from a low ratio in low vehicle speed ranges and present a solution using proposed control. It will also introduce an example of proposed control for high vehicle speed ranges as well.

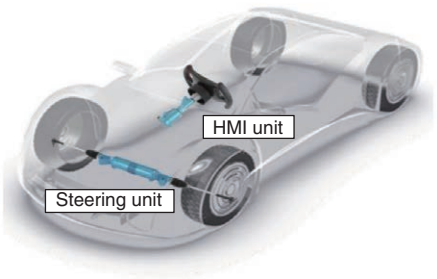

Fig. 1 Steer-by-wire system

2. Issues and Development Goals

2. 1 Issues with Low Ratio

With SBW, the steering ratio can be set without mechanical restriction. For example, by setting the hand wheel angle range to approximately ±150 deg, this will eliminate the need for the driver to change their grip position on the hand wheel in many situations such as when turning at a cross section or when parking in a garage, greatly reducing the operating workloads. At

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the same time, an extremely low ratio will cause too sensitive vehicle behavior in response to hand wheel operation, and the driver will be required for cautious steering in order to reduce this behavior. In other words, lowering the steering ratio not only increases the steady-state gain of yaw rate and lateral acceleration in response to hand wheel angle, but also significantly amplifies the transient response gain to rapid hand wheel operation. **Figure 2** shows the yaw rate response of a test vehicle at speeds of 80 km/h and 20 km/h, and at 20 km/h with a low ratio to set the hand wheel angle ±150 deg range. Comparing the yaw rate response at 80 km/h and 20 km/h, we find that the steady-state gain is higher at 80 km/h. However, the increase ratio of peak gain compared to the steady-state gain is larger at 20 km/h. Additionally, comparing the yaw rate response at 20 km/h with a low ratio, the steady-state gain increases and the peak gain increases more significantly. As a result, rapid hand wheel operation generates a larger yaw rate compared to slow hand wheel operation and increased vehicle behavior gives an effect on maneuverability. It has been pointed out that this kind of large increment of peak gain in low vehicle speed ranges is caused by the tire relaxation length characteristics[4), 5)]. Tire lateral force is generated accompany with the tire rolling movements. So when the vehicle speed is low, the tire rolling speed is low and the delay in this force generation occurs. This leads to a delay in vehicle response and reducing of damping, and the peak gain increases.

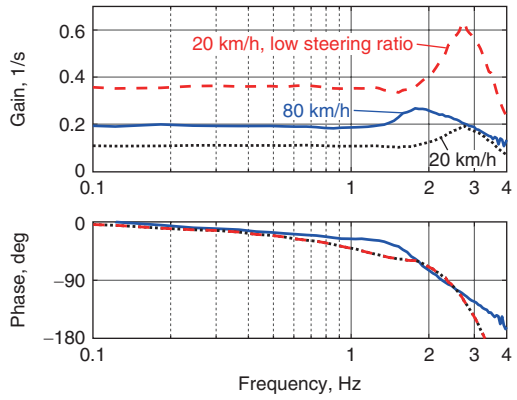

Fig. 2 Yaw rate response for two different speeds

2. 2 Development Goals

If it is possible to reduce peak gain in yaw rate response in low vehicle speed ranges, this may lead to improvement of vehicle behavior and maneuverability when a steering ratio is lowered. Generally, low-pass filters (hereinafter "LPF") are widely used as a process to reduce peak gain. With SBW, the processed input hand wheel angle by the LPF outputs as the road wheel angle. This will be effective to reduce peak gain in the yaw rate response.

Figure 3 shows the yaw rate response at a vehicle speed of 20 km/h. When an LPF is used, the peak gain is reduced. On the other hand, the phase characteristics shows increased delay when an LPF is used. The increase in phase delay means that it takes longer time from the driver's hand wheel operation to the vehicle direction changes, and controllability is an issue in situations that require precise operation such as emergency avoidance and driving on narrow roads. Therefore, if we can achieve characteristics which reduce peak gain without increasing the phase delay (development goal in **Fig. 3**), it should be possible to avoid too sensitive response keeping good maneuverability. In this study, control method that can independently design the frequency response characteristics of gain and phase (hereinafter "proposed control") was developed to achieve these ideal characteristics.

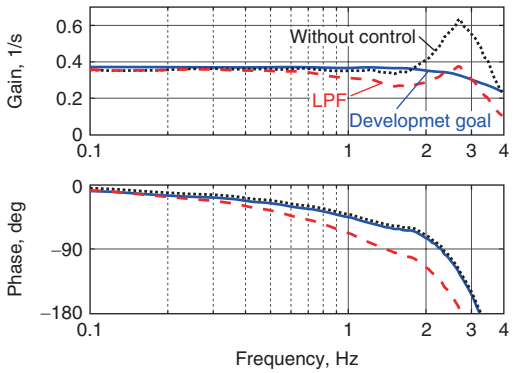

Fig. 3 Frequency response of LPF and development goal

3. Control Structure

The structure of the proposed control is shown in **Fig. 4**. θ is the hand wheel angle input by the driver, γ_T is the target yaw rate, and δ_T is the target road wheel pinion angle. First, the driver hand wheel angle θ that was read from the HMI unit is input into the target yaw rate generator, and γ_T for the set gain characteristics and phase characteristics is output. Next, the front wheel angle calculator calculates the δ_T to achieve the controlled vehicle γ_T. δ_T is input into the steering unit, it steers the tires.

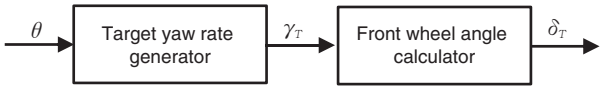

Fig. 4 Control structure

3. 1 Target Yaw Rate Generator

The target yaw rate generator sets the characteristics for yaw rate response gain and phase. The frequency response characteristics output in response to the input are generally expressed by a transfer function, however the gain and phase is related and cannot be set independently. With the proposed control, a neural network is used to set the frequency response characteristics independently for gain and phase. The neural network structure uses the NARX (Non-linear Auto Regressive eXogenous) model[6]. This model is expressed by Formula (1). It is a non-linear auto regressive time series model which expresses the output signal current value $\gamma_T(t)$ using the input signal current value $\theta(t)$ and past values $\theta(t - 1)$, ... , $\theta(t - d)$, and output signal past values $\gamma_T(t - 1)$, ... , $\gamma_T(t - d)$.

$$\gamma_T(t) = f\{\theta(t-1), \cdots \theta(t-d), \gamma_T(t-1), \cdots, \gamma_T(t-d)\} \quad (1)$$

The structure of the target yaw rate generator is shown in **Fig. 5**. The "Training" section is the configuration used for neural network training. The yaw rate response is designed in the phase characteristics setting section and gain characteristics adjustment section. In the phase characteristics setting section, a transfer function is used to set the phase characteristics. The gain characteristics is no need to be specific at this time. The steady-state gain of yaw rate is also set here. In the gain characteristics adjustment section, the gain characteristics are adjusted to the target based on the frequency response characteristics provided by the phase characteristics setting section using a zero-phase filter. The zero-phase filter is an offline filter that performs a filter process of the input data in the forward direction, then reverses the filtering sequence and performs a filter process in the reverse direction, to reduce only the gain without changing the phase. The generated yaw rate response γ_T* is the teacher signal, and the NARX model is trained using this as a target. Input to the NARX model is processed into a multi-dimensional signal with different dynamic characteristics as well as steering angle using a filter group. Based on this, using a variety of input signal behaviors to increase the input information has the effect of increasing the output prediction accuracy. "Delay" in the NARX model is the delay of the input and output and sets the advance number of value which will be used. w is the weight and b is the bias. Training is performed and the constants are adjusted so that error e_N between teacher signal γ_T* and NARX model output γ_T is close to zero. The "Implementation" section is the configuration to be implemented in the on-board ECU. Using the parameters that were decided by the training described above, it is possible to perform online expression of frequency response characteristics which contain offline filter characteristics.

The training results are shown in **Fig. 6**. Here, γ_P is the yaw rate response from the phase characteristics setting section. Compared to γ_P, only the gain characteristics change in γ_T*. Additionally, γ_T* and γ_T show generally consistent results.

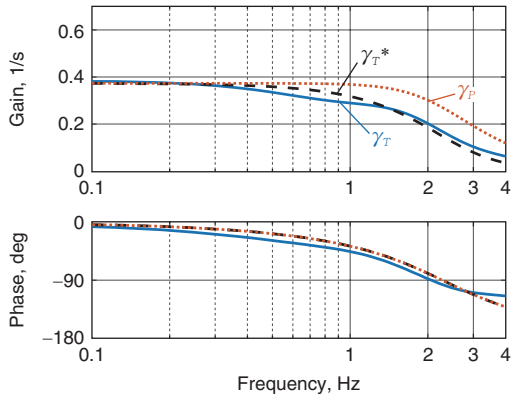

Fig. 6 Frequency response of teacher signal and NARX output

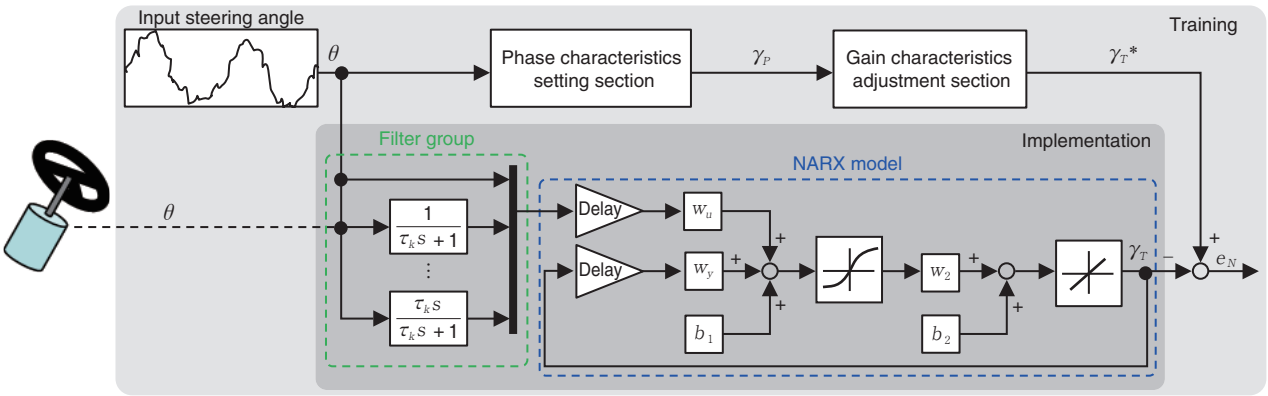

Fig. 5 Target yaw rate generator structure

3. 2 Front Wheel Angle Calculator

The structure of the front wheel angle calculator is shown in **Fig. 7**. In order to reproduce the target yaw rate that was created in the previous section in an actual vehicle, the optimal servo system using the control subject vehicle model calculates the target road wheel pinion angle δ_T. x_1, x_2, and γ_M are state variables, e is the tracking error, and K_1, K_2, and K_e are the feedback gain. As an example, a transfer function for a second order system is used as the vehicle model.

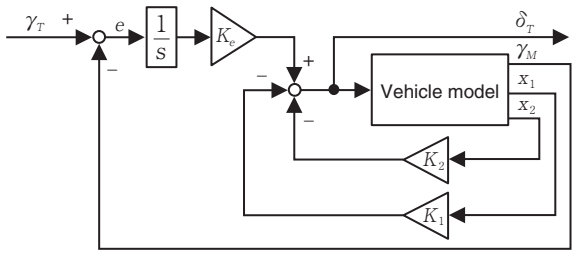

Fig. 7 Front wheel angle calculator structure

Formula (2) shows the vehicle model transfer function, Formula (3) shows the state equation, and Formula (4) shows the output equation. K is the steady-state yaw rate gain, and a_0, a_1, b_0, and b_1 are coefficients.

$$G(s)=\frac{K\cdot(b_1 s+b_0)}{s^2+a_1 s+a_0} \tag{2}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -a_0 & -a_1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ K\end{bmatrix}\delta_T \tag{3}$$

$$\gamma_M=\begin{bmatrix}K\cdot b_0 & K\cdot b_1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \tag{4}$$

e can also be expressed from γ_T and γ_M using Formula (5).

$$e=\gamma_T-\gamma_M \tag{5}$$

Next, when Formula (5) is added to Formula (3) gives Formula (6).

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ e\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ -a_0 & -a_1 & 0\\ -Kb_0 & -Kb_1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ \int e\,dt\end{bmatrix}+\begin{bmatrix}0\\ K\\ 0\end{bmatrix}\delta_T+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}\gamma_T \tag{6}$$

When δ_T is expressed using K_1, K_2, K_e, and e, the result is Formula (7).

$$\delta_T=K_1x_1+K_2x_2+K_e\int e\,dt \tag{7}$$

Finally, we find the K_1, K_2, and K_e that minimize cost function J shown in Formula (8). q_1, q_2, q_e, and R each indicates the weights of x_1, x_2, e, and δ_T.

$$J=\int_0^\infty\left\{\begin{bmatrix}x_1 & x_2 & \int e\,dt\end{bmatrix}\begin{bmatrix}q_1 & 0 & 0\\ 0 & q_2 & 0\\ 0 & 0 & q_e\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ \int e\,dt\end{bmatrix}+R\delta_T{}^2\right\} \tag{8}$$

4. Actual Vehicle Verification

This section shows the effects of the proposed control installed into an SBW test vehicle. It shows an example of the peak gain reduction of the yaw rate response of a vehicle with a low ratio in low vehicle speed ranges and the phase delay reduction in high vehicle speed ranges using this control.

4. 1 Reducing Peak Gain of Yaw Rate Response of Vehicle with Low Ratio in Low Vehicle Speed Ranges

Using the proposed control, the peak gain of the yaw rate response in low vehicle speed ranges is reduced and vehicle stability is improved when the steering ratio is lower. **Table 1** shows the steering system specifications of the test vehicle. ① is a specification in which the hand wheel angle is directly used as the target road wheel pinion angle. From the test vehicle specifications, the hand wheel angle range was ±480 deg. ② is a specification which reduced the steering ratio of ① and set the hand wheel angle range to ±150 deg. Under this specification, changing the grip position on the hand wheel is not required. ③ is a specification which adds the proposed control to the specification in ②.

Table 1 Hand wheel angle range and control type specifications

	Range of steering wheel angle	Control
①	±480 deg	Without
②	±150 deg	Without
③	±150 deg	Proposal

Figure 8 shows the yaw rate response measurement results for each specification at a vehicle speed of 20 km/h. The steady-state gain for ② and ③ was higher than ①, and the peak gain in ② was extremely large. In contrast, the gain in ③ did not increase above the steady-state gain, and the phase maintained approximately the same characteristics as ②.

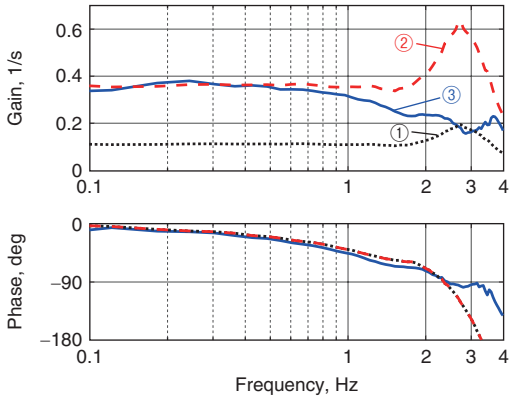

Fig. 8 Yaw rate response of test vehicle

Next, the response for each specification is shown in a time series. **Figure 9** shows the yaw rate measurement results when a sine wave hand wheel angle with frequency varying from 0.5 to 3 Hz was input at 20 km/h. The hand wheel angle had the same amplitude. The yaw rate amplitude in low frequency was the same for ② and ③. However in high frequency, while the yaw rate amplitude increases further for ②, the amplitude decreases for ③. Additionally, the increased timing of yaw rate is almost same as in ① and ② which means the delay does not increase.

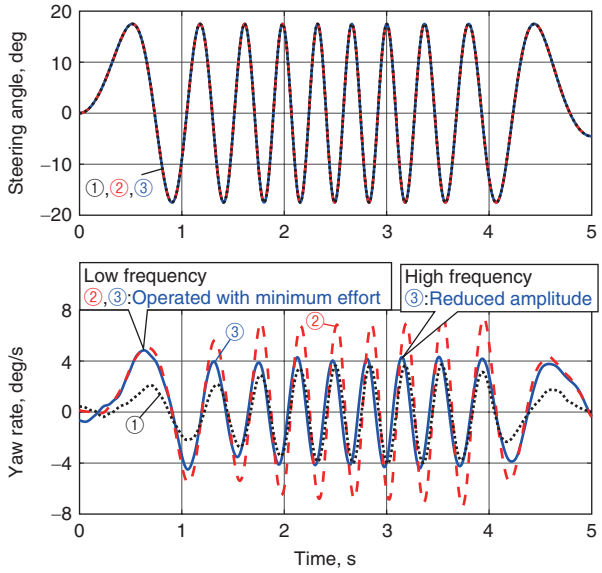

Fig. 9 Time series of input steering angle and yaw rate

4. 2 Reducing Phase Delay in High Vehicle Speed Ranges

This section shows an example of using the proposed control in high vehicle speed ranges. Ordinarily, a smaller phase delay in vehicle behavior contributes higher stability and maneuverability. For example, the addition of the differential term of the input hand wheel angle to the input hand wheel angle can reduce the phase delay[7]. However, focusing on the gain characteristics, peak gain increases with the reduction in phase delay. As a tradeoff to this effect, vehicle behavior becomes larger in high frequency ranges of hand wheel operation, overshoot increases and careful operation is required to prevent it. Using the proposed control, the characteristics that reduce phase delay and maintain the peak gain realizes the improved stability and maneuverability.

Table 2 shows the control specifications of the test vehicle. The steering ratio is the same for all specifications. ④ is a specification in which the hand wheel angle is directly used as the target road wheel pinion angle. ⑤ adds the differential term, and ⑥ is a specification utilizing the proposed control.

Table 2 Control specifications

	Control
④	Without
⑤	With differential term
⑥	Proposal

Figure 10 shows the yaw rate response measurement results for each specification at 80 km/h. While the phase delay is reduced in ⑤ compared to ④, the peak gain is increased. The phase delay in ⑥ is reduced in the same as ⑤, however, the peak gain is approximately the same level as ④.

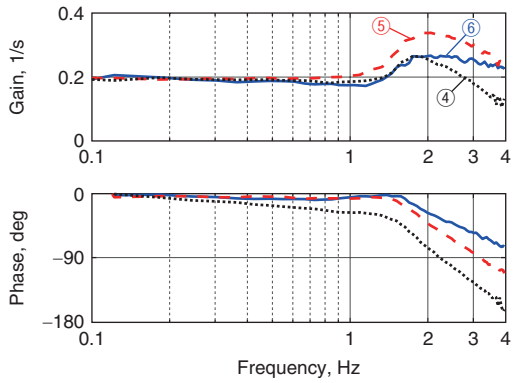

Fig. 10 Yaw rate response of test vehicle

The hand wheel angle during the lane change procedure shown in **Fig. 11** was recorded. Vehicle speed was fixed at 80 km/h, and the driver operates the hand wheel only.

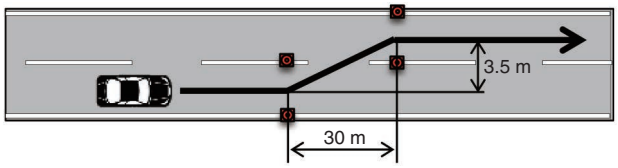

Fig. 11 Lane change procedure

Figure 12 shows the time series data (average of five times driving the course) for hand wheel angle when driving with each specification. ⑤ shows hand wheel operation amplitude and overshoot at the time of convergence are reduced than ④. However the driver comments that the vehicle moved more in response to hand wheel operation and careful operation was required as a result. This means that a tradeoff with increased gain occurs. In case of ⑥, the amplitude was reduced and overshoot at the time of convergence does not occur, this means that stability was improved. The driver also provided positive comments that the amount of required attention to hand wheel operation was reduced compared to ⑤. These results show that reducing the amount of peak gain increment and phase delay during driving in high vehicle speed ranges contributes to improved stability and maneuverability, confirming the effectiveness of the proposed control.

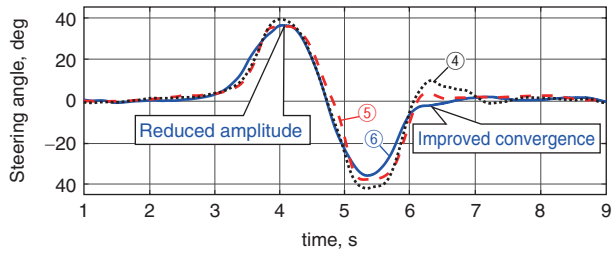

Fig. 12 Hand wheel angle during lane change maneuver

5. Conclusions

This report proposed the control method to adjust the frequency response of yaw rate to the hand wheel angle using SBW. It showed that proposed control method improve stability of vehicle behavior at a low ratio when driving in low vehicle speed ranges, and could contribute to achieving hand wheel operation characteristics that did not require changing the grip position on the hand wheel. In high vehicle speed ranges, the report showed that reduction of the phase delay without increasing the peak gain contributed to improve stability and maneuverability.

In a vehicle with a conventional steering system where the hand wheel and road wheel were mechanically connected, the vehicle frequency response characteristics to the hand wheel operation depended on the specifications of the mechanical components and it was not easy to adjust that characteristics. In SBW vehicles, the proposed control allows characteristics to be designed without changing the mechanical components. This can also contribute to resolving varieties of other issues. We will continue with development aimed at expanding the use of SBW in the future.

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